Discrete Random Structures: Enumeration and Scaling Limits
(SFB Research Network funded by Austrian Science Fund (FWF))
https://SFBrandom.univie.ac.at/

We are currently advertising six PhD positions, associated with the above research network. Random discrete structures, which are ubiquitous in many areas of modern mathematics, are essential to describe a host of phenomena in mathematical physics. For instance they play a key role in our understanding of phase transitions, which describe how physical systems can undergo abrupt changes (as when water changes from liquid to solid state below 0C).

The research network focuses on fundamental two-dimensional examples, including the celebrated dimer model and planar graphs. By combining probabilistic and combinatorial perspectives it is our aim to answer some of the most basic questions about these models: How to enumerate them, either exactly or approximately? How can one describe their random geometry in the large scale limit? How to explain that these structures keep arising under different guises in different problems? Such questions also have deep connections to questions in mathematical physics, from topological phase transitions to Liouville quantum gravity.

The 6 PhD projects are:

1. **Random geometry of dimers and planar maps (Nathanael Berestycki, Network Coordinator, Univ. of Vienna).** The project focuses on the one hand on the dimer model (one of the most classical models of statistical mechanics) and on the other hand on random planar maps, a canonical notion of random surfaces. In both cases we will seek to derive explicit scaling limits which demonstrate the conformal invariance that these models are predicted to acquire in the fine mesh limit. We will also consider the question of the associated critical exponents, which on planar maps are believed to obey the KPZ (Knizhnik--Polyakov--Zamolodchikov) relation of Liouville quantum gravity.

2. **Additive functionals on random discrete structures (Michael Drmota, TU Wien).** The aim of this PhD project is to extend central limit theorems for special additive functionals (like subgraph or pattern counts) to more general additive functionals in the framework of labelled and unlabeled subcritical graphs. More complex models (like planar graphs) may be analyzed, too.

3. **Systematic approaches to bijective proofs (Ilse Fischer, Univ. of Vienna).** The focus is on developing systematic approaches for constructing bijective proofs from known computational proofs. This will be done along several (mostly longstanding) open problems related to the mysterious relation between plane partitions and alternating sign arrays. Among these problems are certain Littlewood-type identities related to alternating sign arrays, which is exciting as this might hint at an Robinson-Schensted-Knuth-type correspondence for alternating sign arrays.

4. **Phase transitions in random combinatorial structures (Mihyun Kang, TU Graz).** This project focuses on the interplay of local and global structure in sparse random planar graphs. The main goals are to determine the threshold for the giant 2-core and the Benjamini–Schramm local weak limit of a random planar graph and to derive the asymptotic number of sparse intercritical planar graphs.

5. **Tiling Enumeration (Christian Krattenthaler, Univ. of Vienna).** This project aims to develop methods for solving (exact and asymptotic) enumerative problems connected with domino tilings and rhombus tilings. In the centre of the project there is Ciucu's electrostatic conjecture on the asymptotic behaviour of the correlation of defects (gaps, holes) as the mutual distances of the defects tends to infinity. It predicts that this asymptotic correlation is given by the (exponential of the negative of) 2D Coulomb energy of the system of charges obtained by regarding each hole as a point charge.

6. **Dimer model: dynamics and scaling limits (Fabio Toninelli, TU Wien).** This PhD project, at the interface between probability theory and mathematical physics, focuses on Markov dynamics of the dimer model and of other statistical physics models. The main goals are the mathematical understanding of mixing properties of the dynamics (speed of convergence to equilibrium, dynamical phase transitions) and of the emergence of large-scale limits (hydrodynamic limits, convergence of the fluctuation process to a stochastic PDE in the continuum limit).

The advertised positions are associated either to the Faculty of Mathematics of the University of Vienna, to the Faculty of Mathematics and Geoinformation of the TU Wien, or to the Faculty of mathematics at the TU Graz. The extent of employment is 30 hours per week with no teaching obligations, and the salary (expected to be around €37.500 gross per year) includes full social security. The funding is guaranteed for at least three years. The positions will start on 01 June 2024 or later.

Application Requirements and Procedure

The basic requirement for the advertised positions is a very good working knowledge in the fields connected to the PhD projects. The candidates must have a master degree (or equivalent) in Mathematics at the moment the PhD position starts. The application should contain:

- a motivation letter, that indicates clearly for which of the 6 PhD projects the candidate is applying; in case of application to more than one of the PhD projects, the letter should indicate an order of preference;
- a scientific CV;
- higher education certificates/diplomas;
- letter(s) of recommendation preferably sent directly to SFBrandom.mathematik@univie.ac.at by the person writing the letter.
- The deadline for applications and for letters is **January 21st, 2024.**

Applications shall be uploaded at

https://SFBrandom.univie.ac.at/open-positions-available/phd-application/