Abstract booklet

Kick-off Event

SFB "Discrete random structures: enumeration and scaling limits"

22-25 September 2024
Hotel Payerbacherhof, Reichenau/Rax

Organisers:
Nathanael Berestycki
Michael Drmota
Ilse Fischer
Mihyun Kang
Christian Krattenthaler
Marcin Lis
Benedikt Stufler
Fabio Toninelli
Astrid Kollros-Spinka

Alternating Sign Matrices, Plane Partitions and their many relatives (Mini-course)

Ilse Fischer
University of Vienna

I will introduce alternating sign matrices and plane partitions as well as their many relatives, such as dimers, six-vertex configurations, bumpless pipe dreams, and tableaux, which make their appearance in several areas, including statistical physics, representation theory, and algebraic geometry. Some of the relations among the various objects are more or less obvious, others are not understood at all, although they are indicated by various equinumerosity phenomena. We will review all that and elaborate on the recent discovery of Littlewood identities and a Cauchy identity, which might indicate that an ASM variation of the famous Robinson-Schensted-Knuth correspondence plays a role.

Introduction to Trees and Planar Maps (Mini-course)

Michael Drmota and Benedikt Stufler
Technical University of Vienna

The mini course aims to introduce the audience to the study of random trees and planar maps. We review selected models and discuss probabilistic and combinatorial aspects such as bijective encodings, asymptotic enumeration, local limits and scaling limits.

Near-critical dimers

Nathanael Berestycki
University of Vienna

We will consider here the dimer model on the square (or hexagonal) lattice with doubly periodic weights, in the near-critical scaling regime. We prove convergence of the height function towards a random field which is not conformally invariant but covariant. In addition, branches in the associated Temperleyan tree converge to Makarov and Smirnov's massive SLE. Time-permitting we will discuss conjectures relating this model to the Sine-Gordon model from quantum field theory.

Scaling limits of random colored trisps

Ariane Carrance
University of Vienna

Colored trisps are a class of discrete spaces that naturally arise as higher-dimensional analogues of maps, in the context of random tensor models. As an approach to quantum gravity, the goal of these models is to obtain relevant continuum spaces as scaling limits of random colored trisps, much like the Brownian sphere is the scaling limit of many models of random planar maps. In this talk, I will first review the results obtained so far by me and others for scaling limits of colored trisps. Secondly, I will present a work in progress with Fabien Vignes-Tourneret, in which we expand known universality classes of random colored trisps, and identify new ones.

Ramanujan's legacies in the world of partitions and q-series

Shane Chen
Technical University of Vienna

In his short but luminous life, Srinivasa Ramanujan, an Indian genius, left us with 21 papers and several short notes, and more importantly, his notebooks containing over 3,900 results, the originality of which has benefited the mathematical world at a substantial level. In this talk, I will give a historical account of Ramanujan's contributions in the world of partitions and q-series, including Ramanujan's congruences, the Rogers-Ramanujan identities, the Hardy-Littlewood-Ramanujan circle method, and the mock theta functions.

Matching problems on sparse random graphs

Mihyun Kang
Technical University of Graz

We will briefly overview classical and recent results concerning matching problems on sparse random graphs. We will also discuss a connection between the matching number of sparse random graphs and the rank of sparse random matrices.

Title: tba

Christian Krattenthaler
University of Vienna

Abstract: tba

Characters of classical groups twisted by roots of unity

Nishu Kumari
University of Vienna

Schur polynomials are the characters of irreducible representations of classical groups of type A parametrized by partitions. For a fixed integer $t >= 2$ and a primitive t'th root of unity ω, the Schur polynomials evaluated at elements $\omega^k x_i$ for $0 = k = t-1$ and $1 = i = n$, were considered by D. J. Littlewood (AMS press, 1950) and independently by D. Prasad (Israel J. Math., 2016). They characterized partitions for which the specialized Schur polynomials are nonzero and showed that if the specialized Schur polynomial is nonzero, it factorizes into characters of smaller classical groups.

In this talk, I will present the Schur factorization result and its various generalizations to the characters of irreducible polynomial representations of the classical groups of type B, C and D.

Zeroes of planar Ising models

Marcin Lis
Technical University of Vienna

In a recent work Livine and Bonzom using physics arguments derived a surprising formula for the Fisher zeroes of the partition function of Ising models

on planar graphs in terms of their embeddings as surfaces in 3D space. I will provide a rigorous proof of this formula (work in progress).

O(n)-invariant quantum spin chains and loop-weighted Lorentz mirror model with n large

Kieran Ryan
Technical University of Vienna

We study a quantum spin chain with two-body interaction and O(n) symmetry. In the ground state, in a large region of the phase diagram and for n large, we prove a translation symmetry breaking. We prove an analogous result for the Lorentz mirror model with loop-weight n, for all the phase diagram and n large enough, using different techniques.

Finally, in all dimensions and at finite (low enough) temperature, we show that the quantum spin system exhibits exponential decay of correlations, again in a large region of the phase diagram and for n large enough. This complements a result of Ueltschi, which says that in dimension at least 3, for n small enough, one has long range order at low temperatures.

Joint work with Jakob Björnberg.

Phase transitions of block-weighted planar maps

Zéphyr Salvy
Technical University of Vienna

Maps can take many different forms: trees, triangulations or even maps with many more edges. Many classes of maps have been enumerated (2-connected maps, trees, quadrangulations, etc.), notably by Tutte, and a phenomenon of universality has been demonstrated: for the majority of them, the number of elements of size n in the class has an asymptotic equivalent of the form $\kappa\rho^{-n}n^{-5/2}$, for a certain κ and a certain ρ. However, there are classes of "degenerate" maps whose behaviour is similar to that of trees, and whose number of elements of size n has an asymptotic equivalent of the form $\kappa\rho^{-n}n^{-3/2}$, such as "outerplanar" maps. This difference in behaviours is observed not only for enumeration, but also for the metric properties. In the "tree" case, the distance between two random vertices is $\sqrt{n}$, compared with $n^{1/4}$ for uniform planar maps of size n. This work focuses on what happens between these two very different regimes. We study a model depending on a parameter $u > 0$ which exhibits the previous

behaviours, and a transition between the two: depending on the position of u with respect to u_C, the behaviour is that of one or other universality class. More precisely, we observe a "subcritical" regime where the scaling limit of the maps is the Brownian map, a "supercritical" regime where it is the Brownian tree and finally a critical regime where it is the 3/2-stable tree. These results are a joint work with Fleurat. Moreover, I show that the method is robust and can be used to study a variety of similar models. In particular, in a joint work with Albenque and Fusy, we studied tree-rooted random planar maps decomposed into tree-rooted 2-connected blocks, where a spanning tree is drawn simultaneously with the map. This model, which is of interest in theoretical physics, shows new behaviours.

Independent sets in (hyper)graphs and related problems

Mihalis Sarantis
Technical University of Graz

Independent sets (IS) in graphs and hypergraphs is a structure that has attracted great interest in combinatorics, computer science and statistical physics. In this talk, we will review some of the related literature and certain well-studied questions. Given structural properties of a (hyper)graph (such as max-degree, uniformity or regularity) can we bound the number of IS? Which are the extremal (hyper)graphs, if they exist? Can we approximately count the number of IS efficiently? Central to the study of these questions is the independence polynomial and its closely related probabilistic model, the hard-core model.

On the other hand, statistical physicists are very interested in the behavior of models on infinite graphs (particularly lattices), such as long-range correlations and equilibrium states. This requires the definition of Gibbs measures and the study of its properties.

Finally, we will discuss connections between these questions, as well as interesting open problems and future directions.

Mixing time of random tilings

Fabio Toninelli
Technical University of Vienna

I will discuss rhombus tilings of a (tilable) finite subset D of the plane and Markov chains (Glauber dynamics) that are reversible with respect to the uniform measure over all possible tilings. What is the mixing time T_{mix}? Under

some natural conditions on the domain D, it is expected that T_{mix} grows like $L^{2+o(1)}$ in continuous time, where L is the diameter of D. I will discuss recent and less recent results in this direction, as well as some intriguing open problems (based on joint works with Benoit Laslier, and on ongoing work with Amol Aggarwal).

Degree conditions for independent transversals

Ronen Wdowinski
Technical University of Graz

Given a graph G and a partition P of its vertex set, an independent transversal is an independent set of G that contains one vertex from each class of P. In this talk, we will describe a topological Hall-type theorem, due to Aharoni, Haxell (2000) and Meshulam (2003), that gives a sufficient condition for the existence of an independent transversal in terms of the topological connectedness of the independence complex of the graph. We will then explain how to use this theorem to derive degree conditions for independent transversals. In particular, we will use it to find an interpolation between an optimal maximum degree condition of Haxell (1995, 2001) and an optimal average degree condition of Wanless, Wood (2022), which fully answers a question of Groenland, Kaiser, Treffers, Wales (2023). This is based on joint work with Penny Haxell.
